Enhancing energy harvesting by coupling monostable oscillators

J.I. Peña Rosselló[1], H.S. Wio[2], R.R. Deza[1], P. Hänggi[3]

[1] IFIMAR (UNMdP-CONICET) Mar del Plata, Argentina
[2] IFCA (UC-CSIC) Santander, Spain
[3] Universität Augsburg & Nanosystems Initiative Munich, Germany

Gubbio, July 4, 2017

Abstract

- The performance of a ring of linearly coupled, monostable nonlinear oscillators is optimized towards its goal of acting as energy harvester—through piezoelectric transduction—of mesoscopic fluctuations, which are modeled as Ornstein–Uhlenbeck noises.
- For a single oscillator, the maximum output voltage and overall efficiency are attained for a soft piecewise-linear potential (providing a weak attractive constant force) but they are still fairly large for a harmonic potential.

Abstract (cont.)

- When several harmonic springs are linearly and bidirectionally coupled to form a ring, it is found that counter-phase coupling can largely improve the performance while in-phase coupling worsens it.
- Moreover, it turns out that few (two or three) coupled units perform better than more.

Introduction

- This work continues the study of coupling effects initiated in Deza *et al.*, 2013.
- The model oscillator (proposed as piezoelectric energy harvester) is assumed to obey a bounded monostable potential of the form $U(x) = a_n |x|^n$.
- First, explore the relationship between the system's performance and parameters a_n, n.

Introduction (cont.)

- Next, influence on system's performance of linearly and bidirectionally coupling a set of harmonic oscillators: we optimize the sign and strength of the coupling and the number of coupled oscillators.
- Finally, effect on optimal configuration of further adjusting parameter a_n (with the condition $a_n < 1$).

The model

One-dimensional inertial nonlinear oscillator $x(t)$ with mass m and damping constant γ, governed by monostable potential $U(x)$.

Coupled with strength σ to a source of mechanical vibrations that produces an instantaneous force $\xi(t)$, and to a piezoelectric transducer.

This provides voltage $V(t) = K_c\, x(t)$ and reacts back on oscillator with force $K_v\, V(t)$.

The model (cont.)

Output voltage fed into a load circuit, with resistance R and capacitance C (time constant $\tau_p = RC$).

$$m\ddot{x} = -U'(x) - m\gamma\,\dot{x} - K_v V + \sigma\,\xi(t), \quad (1)$$

$$\dot{V} = K_c\,\dot{x} - V/\tau_p. \quad (2)$$

Source of mechanical vibrations $\xi(t)$—regarded as stochastic but self-correlated or "colored"—modeled as Ornstein–Uhlenbeck noise with zero mean and self-correlation function

$$\langle \xi(t)\xi(t') \rangle = \tau^{-2} \exp[(t - t')/\tau].$$

The model (cont.)

One of our goals is to infer the oscillator potential that maximizes efficiency. Restrict search to

$$U(x) = a_n\,|x|^n,$$

which become analytic for even n.

Here $a_n = U_0/|x_0|^n$, where U_0 has energy units and x_0 is a characteristic length, which can be taken as

$$\sqrt{\int \mathrm{d}x\, x^2 \exp[-U(x)/\sigma^2]}.$$

The model (cont.)

Measures of performance:

- rms output voltage

$$V_{\mathrm{rms}} := \langle V^2 \rangle^{1/2}$$

$\langle V^2 \rangle$ implies time-average during observation interval and ensemble-average over noise realizations,

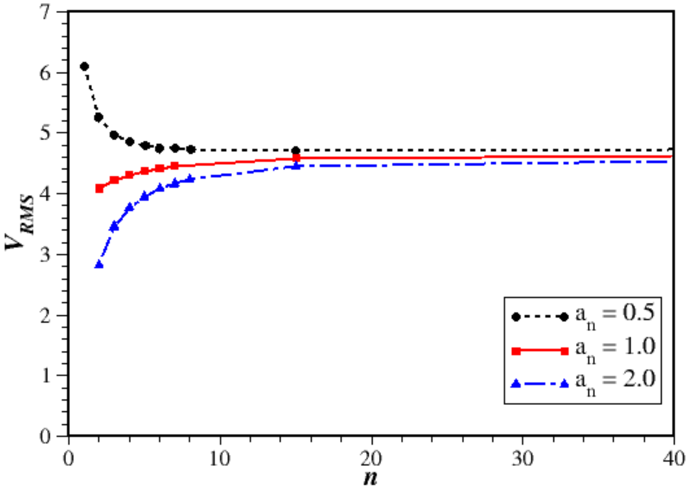
- efficiency

$$\eta = \eta_{\mathrm{me}} \eta_{\mathrm{nm}} = \frac{1}{R} \frac{\langle V^2 \rangle}{\langle \dot{x}\, \xi \rangle},$$

η_{me}: transducer efficiency (mechanical to electric)
η_{nm}: oscillator efficiency (external noise to mechanical).

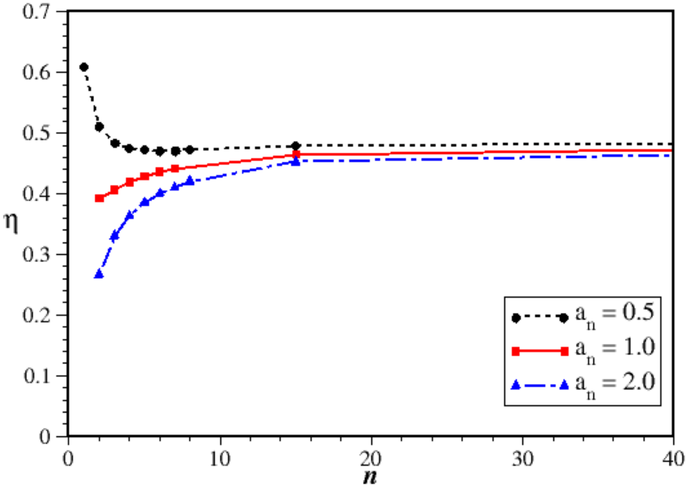
V_{rms} vs n

$m = 1$, $\gamma = 1$, $K_v = 1$, $K_c = 1$, $\sigma = 1$, $\tau = 1$, $\tau_p = 2$.



η vs n

$m = 1$, $\gamma = 1$, $K_v = 1$, $K_c = 1$, $\sigma = 1$, $\tau = 1$, $\tau_p = 2$.



Role of exponent n

With their own peculiarities, both performance indicators follow the same trends:

a) strong (very weak) dependence on n for its lowest (higher) values,

b) performance improvement (worsening) at low n for $a_n < 1$ ($a_n \geq 1$).

In agreement with Gammaitoni *et al.*, 2010, the best performance ($V_{\mathrm{rms}} = 6.107$, $\eta = 0.609$) is attained with the lowest n and a_n in this set, namely

$$U(x) \propto \frac{1}{2}|x|.$$

Note however that for $n = 2$ (and $a_n = 0.5$) both V_{rms} and η are fairly high, possessing the additional advantage of the potential being analytic.

Coupling N units with coupling strength k

Next step in optimizing the device: to find most suitable

- linear coupling between units,
- number of units,

assuming periodic boundary conditions.

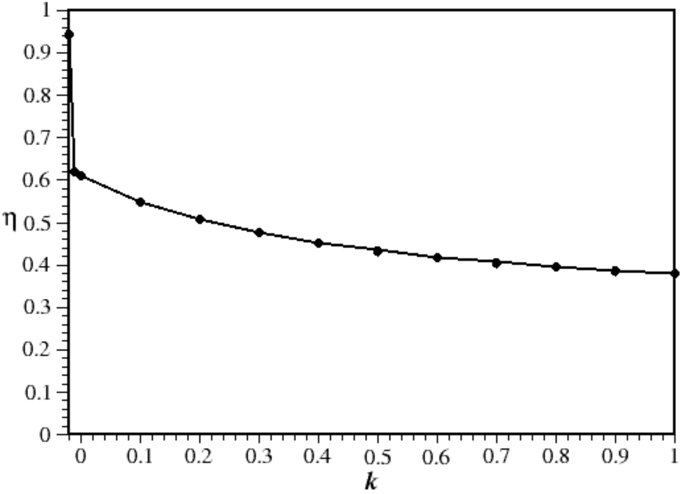
We delve on preliminary evidence Deza *et al.*, 2013 that counter-phase (anti-diffusive or anti-ferromagnetic, $k < 0$) coupling outperforms in-phase (diffusive or ferromagnetic, $k > 0$) one.

Simplest generalization of Eqs. (1)–(2) is

$$
\begin{aligned}
m\,\ddot{x}_j &= -U'(x_j) - m\gamma\,\dot{x}_j - K_v V_j + \sigma\,\xi_j(t) + k(x_{j+1} - 2x_j + x_{j-1}),\\
\dot{V}_j &= K_c\,\dot{x}_j - V_j/\tau_p, \qquad j = 1, \ldots, N.
\end{aligned}
$$

η vs k, for $n = 1$

As k becomes slightly negative, η (and V_{rms}, not shown) diverge.



...coupling strength k (cont.)

For $U(x) \propto \frac{1}{2}|x|$, the performance indicators diverge for $k_{\rm di} \approx -0.02$ (for $k = -0.01$, η scales up to 0.62 and $V_{\rm rms}$ reaches 6.23).

This divergence indicates a diffusive instability, more clearly seen by writing

$$m\,\ddot{x}_j = -U'_{\rm eff} - m\gamma\,\dot{x}_j - K_v V_j + \sigma\,\xi_j(t)$$

with

$$U_{\rm eff} = \sum_j \left\{ U(x_j) + \frac{k}{2}\left[(x_{j+1} - x_j)^2 + (x_j - x_{j-1})^2\right]\right\},$$

which has the continuous form

$$U_{\rm eff} = \int \mathrm{d}l \left\{ U(x(l)) + \frac{k}{2}(\partial_l x)^2 \right\}.$$

... coupling strength k (cont.)

For $\sigma = 0$, $k > 0$ favors in-phase oscillation (uniform ground state) whereas $k < 0$ favors counter-phase oscillation (finite-wavevector ground state).

If by effect of noise, a given oscillator performs a large excursion (so producing a large piezoelectric voltage), $k > 0$ (diffusive coupling) will tend to smooth it up, whereas $k < 0$ (antidiffusive coupling) will tend to enhance it.

A deeper analysis with $U(x) \propto \frac{1}{2}|x|$ requires higher-order corrections (terms beyond the diffusive one) in the definition of the coupling.

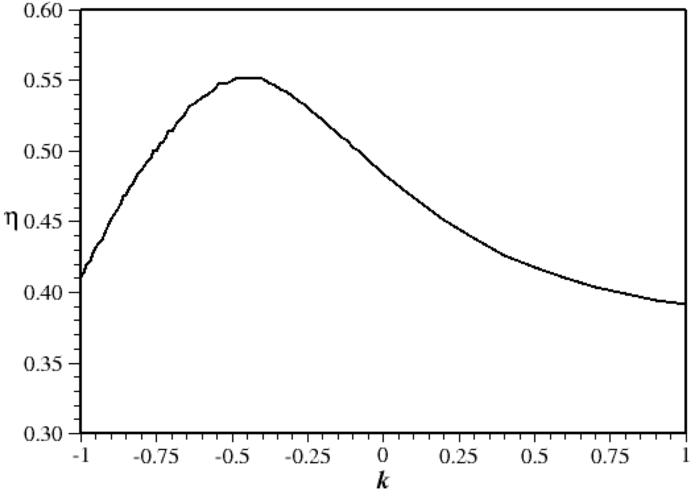
. . . coupling strength k (cont.)

An issue worth exploring is the dependence of k_{di} on n, given that the uncoupled performance is larger for the lowest n values than for the rest, being they still very good for $n = 2$.

In the coupled case, no diffusive instability shows up for $n > 2$. Already for $n = 3$, a maximum is observed in the performance indicators at some $k_{\mathrm{max}} \approx -0.5$, with $\eta = 0.552$ and $V_{\mathrm{rms}} = 5.492$ (not shown). For $n = 4$, $\eta = 0.508$ and $V_{\mathrm{rms}} = 5.074$.

There is still a diffusive instability for $n = 2$ and $N = 3$, but k_{di} is shifted toward safely larger values, allowing η and V_{rms} (not shown) to largely exceed the maximum values attained for $n > 2$.

For $n > 2$, no divergence shows up in either η or $V_{\rm rms}$ for $k < 0$



Number N of units ($n = 2$)

After confirming preliminary evidence that the performance of antidiffusively coupled units can be notably enhanced w.r.t. the uncoupled case, the question is what's its optimal number N.

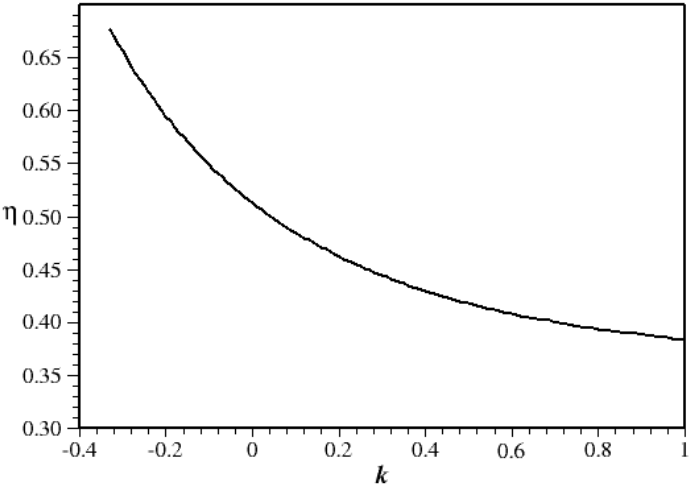
For our surprise, best performance is attained for very low N (two or three). Afterwards, a kind of plateau is reached.

Configurations with odd N perform (slightly) better than those with even N, because they yield on average a net displacement.

As expected, $k_{\max}$ depends on N.

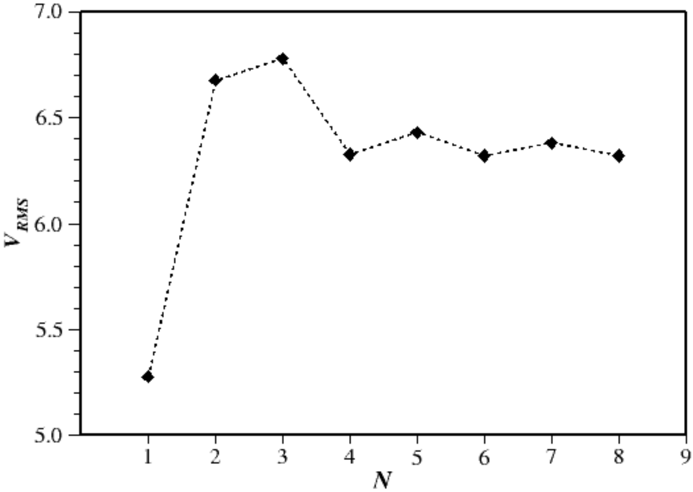
Three coupled harmonic oscillators

Divergence shows up at larger $|k|$; $\eta = 0.677$ and $V_{\rm rms} = 6.778$ for $k = -0.33$



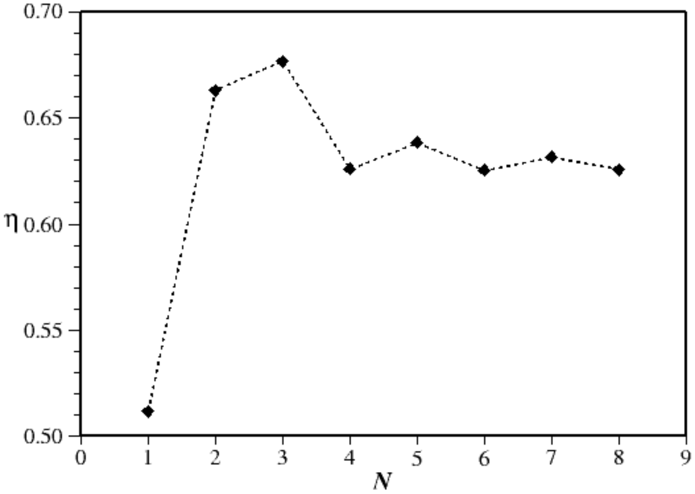
V_{rms} vs N ($n = 2$)

$m = 1$, $\gamma = 1$, $K_v = 1$, $K_c = 1$, $\sigma = 1$, $\tau = 1$, $\tau_p = 2$



η vs N ($n = 2$)

$m = 1$, $\gamma = 1$, $K_v = 1$, $K_c = 1$, $\sigma = 1$, $\tau = 1$, $\tau_p = 2$



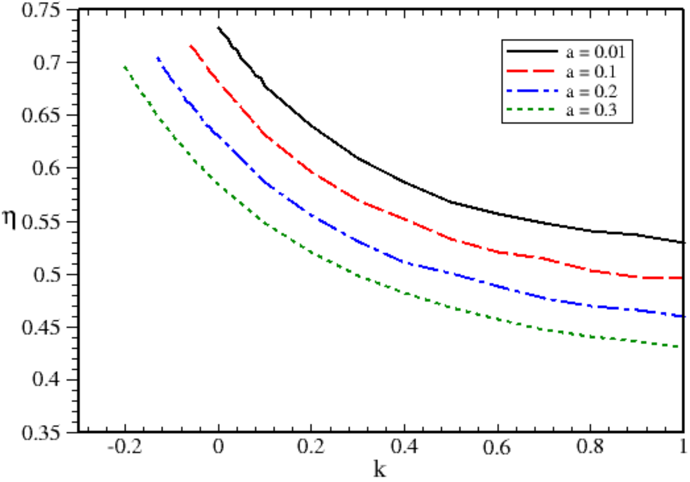
Role of the potential's softness

Once the optimal configuration ($N = 3$, $a_n < 1$) has been found for $n = 2$, we decrease a_n further in order to check whether the performance keeps improving, and find the minimum value of a_n for which the negative couplings lose meaning (without considering higher order corrections).

We have found that $|k_{\mathrm{di}}|$ decreases as a_n does. The limit seems to be around $a_n = 0.01$, beyond which every negative coupling between units leads to a divergence in η and V_{rms} (clearly, a small a_n implies either a small U_0 or a large x_0).

η vs k for different a_n.

$m = 1$, $\gamma = 1$, $K_v = 1$, $K_c = 1$, $\sigma = 1$, $\tau = 1$, $\tau_p = 2$.



Conclusions

For a model oscillator proposed as an energy harvester through piezoelectric conversion, we have analyzed monostable potentials of the form $U(x) = a_n |x|^n$.

In agreement with Gammaitoni *et al.*, 2010, the best performance is attained with $n = 1$ and $a_n < 1$; namely a soft piecewise-linear potential, providing a weak attractive constant force.

The performance indicators strongly decay with n for its few lowest values, but they are still high enough for the harmonic oscillator.

Conclusions (cont.)

We have studied the effect of coupling between units and sought the optimal number N of coupled units in order to enhance the system's energy harvesting.

In agreement with Deza *et al.*, 2013, whilst diffusive coupling between units reduces the system's performance, anti-diffusive couplings cause an enhancement.

Next we found that it does so via the mechanism of a diffusive instability. As a metaphor, we can picture this situation as an anti-ferromagnetic coupling between spins, or an inhibitory coupling in neuron systems.

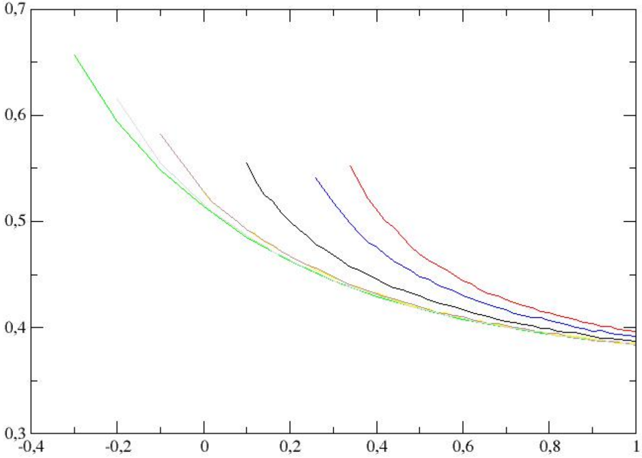
Conclusions (cont.)

The main point is how to implement such a form of coupling. The foregoing results are just indicative, and should be regarded as a “toy model”.

However, even such a simple setup depicts the fact that corrections going beyond a diffusive coupling need to be added in order to globally control the instability. This will be subject of further studies, together with more elaborate aspects and/or models.

Further developments

additional KPZ term. η vs *k*



Thanks!

References

- J.I. Deza, R.R. Deza, and H.S. Wio, Nanoenergy Lett. **6**, 29 (2013)
- L. Gammaitoni, I. Neri, and H. Vocca, Chem. Phys. Lett. **375**, 435 (2010)
- J.I. Peña Rosselló, H.S. Wio, R.R. Deza, and P. Hänggi, Eur. Phys. J. B **90**, 34 (2017)